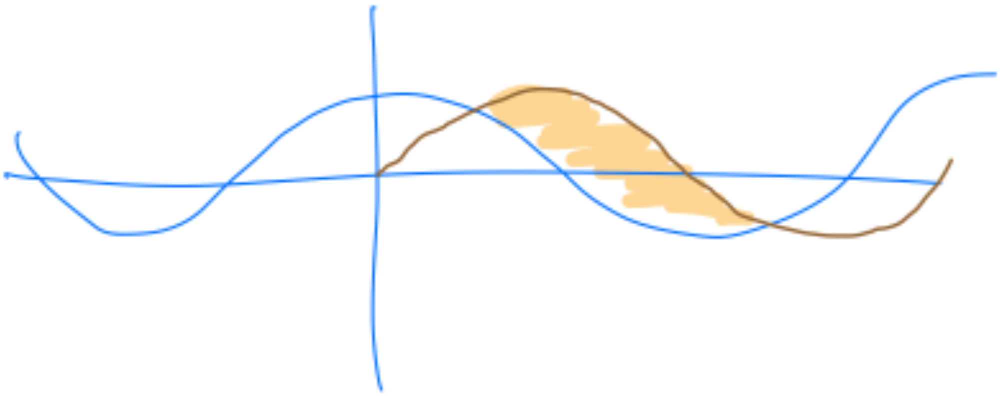
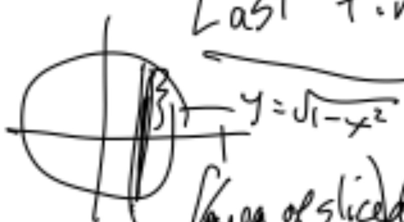


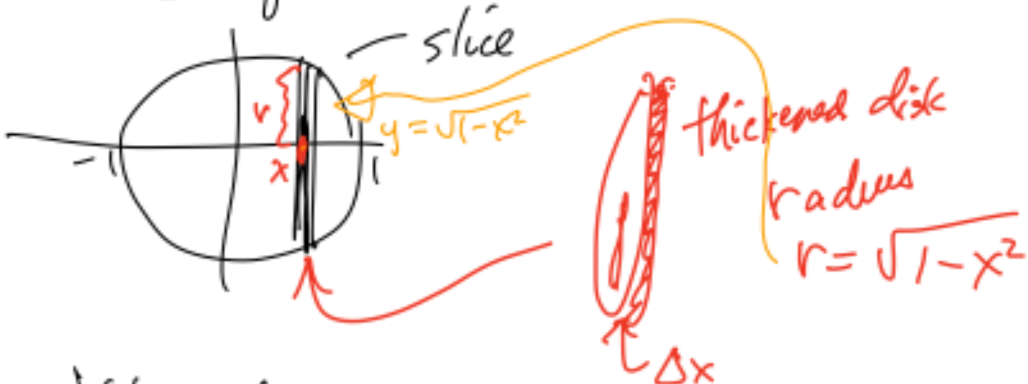
Area of Disk — 2-d

Last time: Area of unit disk = π ← radius.



$\int_{-1}^{1} (\text{area of slice})\, dx$ $\Rightarrow$ Area of disk of radius R $= \pi R^2$.

~~Area~~ Volume of unit Ball in 3-d

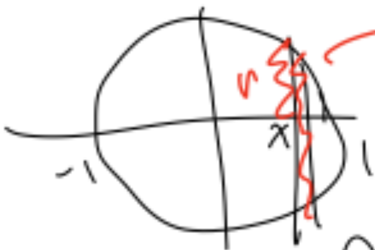


$$\text{Volume of unit ball} = \int_{-1}^{1} \underbrace{\pi \big(\overbrace{\sqrt{1-x^2}}^{r}\big)^2}_{\text{2d area of disk of radius } \sqrt{1-x^2}} dx$$

$$\ldots\ldots = \frac{4\pi}{3}$$ volume of unit ball in 3-d.

$$\Rightarrow \left(\text{Volume of 3-d ball of radius } R\right) = \frac{4\pi}{3}R^3$$

4-d Ball   unit ball



slice = 3-d ball of radius $r=\sqrt{1-x^2}$ thickened by Δx.

3-d volume of ~~ball of radius r~~   thickness

$$\begin{pmatrix}\text{4-D} \\ \text{Volume} \\ \text{unit} \\ \text{4-ball}\end{pmatrix} = \int_{-1}^{1} \underbrace{\frac{4\pi}{3}\left(\sqrt{1-x^2}\right)^3}_{} \, dx \quad \cdots \quad \frac{\pi^2}{2}$$

$$\left(\text{Volume of a ball of radius } R\right) = \frac{\pi^2}{2} R^4$$

SageMathCell

Type some Sage code below and press Evaluate.

```
F(x)=3^x
g(x)=x^3
a = plot(F(x),(x,-1,4))
b = plot(g(x),(x,-1,5))
#show(a+b)
h(x)=F(x)-g(x)
c=plot(h(x),(x,-1,4))
show(c)
find_root(h(x),2.1,2.9)
```

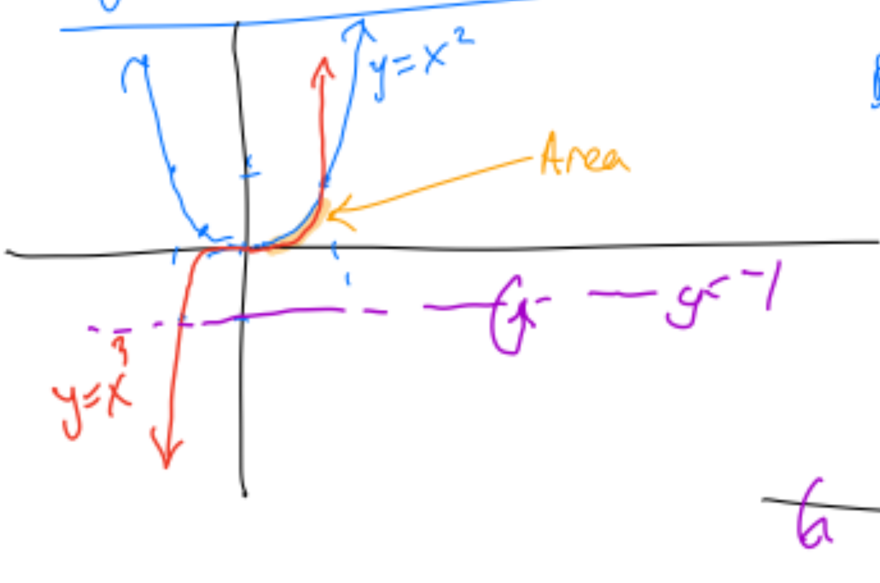
Evaluate

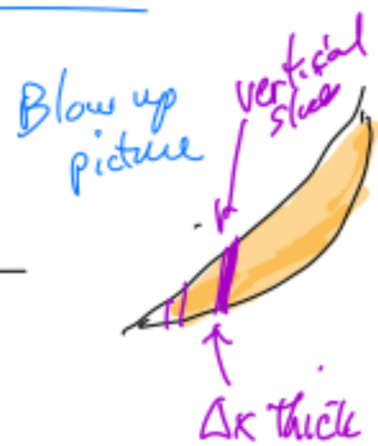
More examples of finding volume of objects

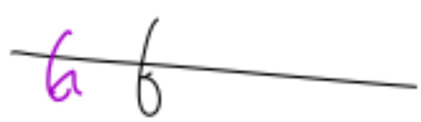
By revolving areas around a line, we can construct 3-d objects. — can use integrals to find the volume.

$$\text{Vol} = \int (\text{volume of slice})(\text{thickness})\ \begin{matrix} dx \\ dy \end{matrix}$$

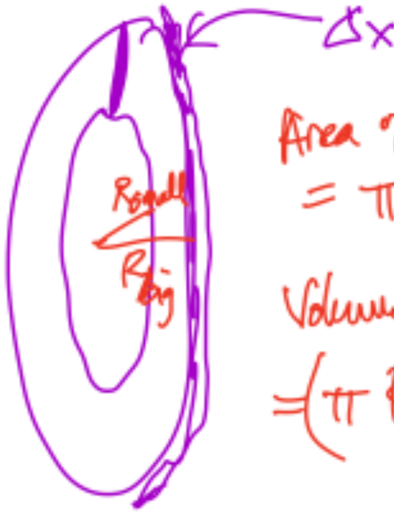
<u>Example</u>: Suppose the area between $y = x^3$ and $y = x^2$ in the first quadrant is revolved around the line $y=-1$. What is the volume generated?





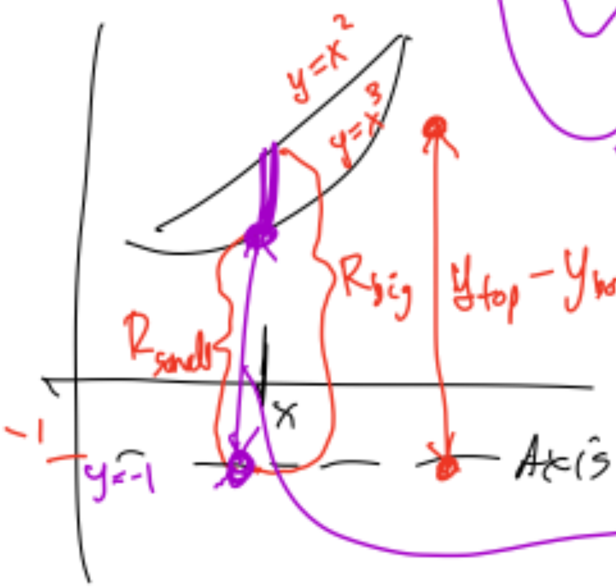


Shape of slice = washer



Area of washer $= \pi\left(R_{big}^2\right) - \pi\left(R_{small}^2\right)$

Volume of washer $= \left(\pi R_{big}^2 - \pi R_{small}^2\right)\Delta x$



$y_{top} - y_{bottom} = x^2 - (-1) = x^2+1$

$(y_{top} - y_{bottom})$

$R_{small} = x^3 - (-1) = (x^3+1)$

Volume of slice $= \left(\pi R_{big}^2 - \pi R_{small}^2\right)\Delta x$

$$= \left[\pi(x^2+1)^2 - \pi(x^3+1)^2\right]\Delta x$$

$$\text{Total volume} = \int_{x=0}^{1} \left(\pi(x^2+1)^2 - \pi(x^3+1)^2\right) dx$$

$= \ldots =$

2:48 PM Mon Feb 27

sagecell.sagemath.org

Type some Sage code below and press Evaluate.

```
f(x)=pi*(x^2+1)^2-pi*(x^3+1)^2
ii = integral(f(x),(x,0,1))
show(ii)
```

Evaluate

$$\frac{47}{210}\,\pi$$